## Solutions of the problems of the IV International Olympiad, Moscow, 1970
## Theoretical Competition

**Problem 1.**

a) By the moment of releasing the bar the carriage has a velocity $v_0$ relative to the table and continues to move at the same velocity.

The bar, influenced by the friction force $F_{fr} = \mu mg$ from the carriage, gets an acceleration $a = F_{fr}/M = \mu mg/M$; $a = 0.02$ m/s , while the velocity of the bar changes with time according to the law $v_b = at$.

Since the bar can not move faster than the carriage then at a moment of time $t = t_0$ its sliding will stop, that is $v_b = v_0$. Let us determine this moment of time:

$$t_0 = \frac{v_0}{a} = \frac{v_0 M}{\mu mg} = 5\text{s}$$

By that moment the displacement of the *Sb* bar and the carriage *Sc* relative to the table will be equal to

$$S_c = v_0 t_0 = \frac{v_0^2 M}{\mu mg}, \quad S_b = \frac{a t_0^2}{2} = \frac{v_0^2 M}{2\mu mg}.$$

The displacement of the carriage relative to the bar is equal to

$$S = S_c - S_b = \frac{v_0^2 M}{2\mu mg} = 0.25\text{m}$$

Since $S<l$, the carriage will not reach the edge of the bar until the bar is stopped by an immovable support. The distance to the support is not indicated in the problem condition so we can not calculate this time. Thus, the carriage is moving evenly at the velocity $v_0 = 0.1$ m/s, whereas the bar is moving for the first 5 sec uniformly accelerated with an acceleration $a = 0.02$ m/s and then the bar is moving with constant velocity together with the carriage.

b) Since there is no friction between the bar and the table surface the system of the bodies "bar-carriage" is a closed one. For this system one can apply the law of conservation of momentum:

$$mv + Mu = mv_0 \qquad (1)$$

where $v$ and $u$ are projections of velocities of the carriage and the bar relative to the table onto the horizontal axis directed along the vector of the velocity $v_0$. The velocity of the thread winding $v_0$ is equal to the velocity of the carriage relative to the bar ($v$-$u$), that is

$$v_0 = v - u \qquad (2)$$

Solving the system of equations (1) and (2) we obtain:

$$u = 0, \quad v = v_0.$$

Thus, being released the bar remains fixed relative to the table, whereas the carriage will be moving with the same velocity $v_0$ and will reach the edge of the bar within the time $t$ equal to

$$t = l/v_0 = 5 \text{ s}.$$