Problem 3. A small charged ball of mass m and charge q is suspended from the highest point of a ring of radius R by means of an insulating cord of negligible mass. The ring is made of a rigid wire of negligible cross section and lies in a vertical plane. On the ring there is uniformly distributed charge Q of the same sign as q. Determine the length l of the cord so as the equilibrium position of the ball lies on the symmetry axis perpendicular to the plane of the ring.

Find first the general solution a then for particular values $Q = q = 9.0 \cdot 10^{-8}$ C, $R = 5$ cm, $m = 1.0$ g, $\varepsilon_0 = 8.9 \cdot 10^{-12}$ F/m.

Solution:
In equilibrium, the cord is stretched in the direction of resultant force of $\vec{G} = m\vec{g}$ and $\vec{F} = q\vec{E}$, where $\vec{E}$ stands for the electric field strength of the ring on the axis in distance x from the plane of the ring, see Figure 3. Using the triangle similarity, one can write

$$\frac{x}{R} = \frac{Eq}{mg}\,. \tag{11}$$

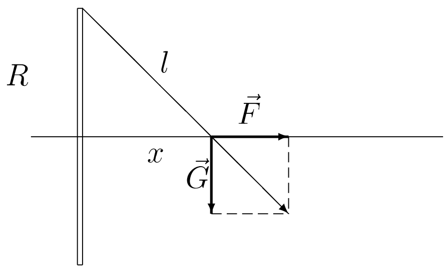

Figure 3:

For the calculation of the electric field strength let us divide the ring to n identical parts, so as every part carries the charge Q/n. The electric field strength magnitude of one part of the ring is given by

$$\Delta E = \frac{Q}{4\pi\varepsilon_0 l^2 n}\,.$$

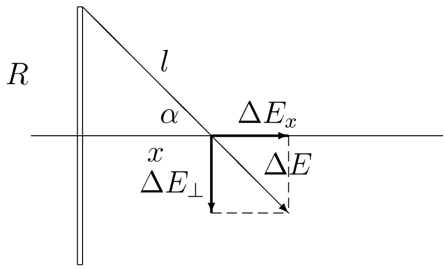

Figure 4:

This electric field strength can be decomposed into the component in the direction of the x-axis and the one perpendicular to the x-axis, see Figure 4. Magnitudes of both components obey

$$\Delta E_x = \Delta E \cos\alpha = \frac{\Delta E\, x}{l},$$
$$\Delta E_\perp = \Delta E \sin\alpha .$$

It follows from the symmetry, that for every part of the ring there exists another one having the component $\Delta\vec{E_\perp}$ of the same magnitude, but however oppositely oriented. Hence, components perpendicular to the axis cancel each other and resultant electric field strength has the magnitude

$$E = E_x = n\Delta E_x = \frac{Q\,x}{4\pi\varepsilon_0\, l^3} . \tag{12}$$

Substituting (12) into (11) we obtain for the cord length

$$l = \sqrt[3]{\frac{Q\,q\,R}{4\pi\varepsilon_0\, m\, g}} .$$

Numerically

$$l = \sqrt[3]{\frac{9.0\cdot 10^{-8}\cdot 9.0\cdot 10^{-8}\cdot 5.0\cdot 10^{-2}}{4\pi\cdot 8.9\cdot 10^{-12}\cdot 10^{-3}\cdot 9.8}}\ \mathrm{m} = 7.2\cdot 10^{-2}\ \mathrm{m} .$$

Problem 4. A glass plate is placed above a glass cube of 2 cm edges in such a way that there remains a thin air layer between them, see Figure 5.