Using the last result we can calculate

$$a = a_x = -a_y = \frac{m_1}{m_1 + m_2}\, g\,,$$
$$T_2 = T_1 = \frac{m_2 m_1}{m_1 + m_2}\, g\,.$$

Numerical results:

$$a = a_x = \frac{3}{5} \cdot 9.81 \text{ m s}^{-2} = 5.89 \text{ m s}^{-2}\,,$$
$$T_1 = T_2 = 1.18 \text{ N}\,.$$

Problem 2. Water of mass m_2 is contained in a copper calorimeter of mass m_1. Their common temperature is t_2. A piece of ice of mass m_3 and temperature $t_3 < 0\,^{\circ}\mathrm{C}$ is dropped into the calorimeter.

a) Determine the temperature and masses of water and ice in the equilibrium state for general values of m_1, m_2, m_3, t_2 and t_3. Write equilibrium equations for all possible processes which have to be considered.

b) Find the final temperature and final masses of water and ice for $m_1 = 1.00$ kg, $m_2 = 1.00$ kg, $m_3 = 2.00$ kg, $t_2 = 10\,^{\circ}\mathrm{C}$, $t_3 = -20\,^{\circ}\mathrm{C}$.

Neglect the energy losses, assume the normal barometric pressure. Specific heat of copper is $c_1 = 0.1$ kcal/kg·°C, specific heat of water $c_2 = 1$ kcal/kg·°C, specific heat of ice $c_3 = 0.492$ kcal/kg·°C, latent heat of fusion of ice $l = 78,7$ kcal/kg. Take 1 cal = 4.2 J.

Solution:
We use the following notation:

t	temperature of the final equilibrium state,
$t_0 = 0\,^{\circ}\mathrm{C}$	the melting point of ice under normal pressure conditions,
M_2	final mass of water,
M_3	final mass of ice,
$m_2' \leq m_2$	mass of water, which freezes to ice,
$m_3' \leq m_3$	mass of ice, which melts to water.

a) Generally, four possible processes and corresponding equilibrium states can occur:

1. $t_0 < t < t_2$, $m_2' = 0$, $m_3' = m_3$, $M_2 = m_2 + m_3$, $M_3 = 0$.
   Unknown final temperature t can be determined from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t) = m_3c_3(t_0 - t_3) + m_3l + m_3c_2(t - t_0)\,. \qquad (7)$$

   However, only the solution satisfying the condition $t_0 < t < t_2$ does make physical sense.

2. $t_3 < t < t_0$, $m_2' = m_2$, $m_3' = 0$, $M_2 = 0$, $M_3 = m_2 + m_3$.
   Unknown final temperature t can be determined from the equation

$$m_1c_1(t_2 - t) + m_2c_2(t_2 - t_0) + m_2l + m_2c_3(t_0 - t) = m_3c_3(t - t_3)\,. \quad (8)$$

   However, only the solution satisfying the condition $t_3 < t < t_0$ does make physical sense.

3. $t = t_0$, $m_2' = 0$, $0 \le m_3' \le m_3$, $M_2 = m_2 + m_3'$, $M_3 = m_3 - m_3'$.
   Unknown mass m_3' can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) = m_3c_3(t - t_3) + m_3'l\,. \qquad (9)$$

   However, only the solution satisfying the condition $0 \le m_3' \le m_3$ does make physical sense.

4. $t = t_0$, $0 \le m_2' \le m_2$, $m_3' = 0$, $M_2 = m_2 - m_2'$, $M_3 = m_3 + m_2'$.
   Unknown mass m_2' can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) + m_2'l = m_3c_3(t_0 - t_3)\,. \qquad (10)$$

   However, only the solution satisfying the condition $0 \le m_2' \le m_2$ does make physical sense.

b) Substituting the particular values of m_1, m_2, m_3, t_2 and t_3 to equations (7), (8) and (9) one obtains solutions not making the physical sense (not satisfying the above conditions for t, respectively m_3'). The real physical process under given conditions is given by the equation (10) which yields

$$m_2' = \frac{m_3c_3(t_0 - t_3) - (m_1c_1 + m_2c_2)(t_2 - t_0)}{l}\,.$$

Substituting given numerical values one gets $m_2' = 0.11$ kg. Hence, $t = 0\,^\circ$C, $M_2 = m_2 - m_2' = 0.89$ kg, $M_3 = m_3 + m_2' = 2.11$ kg.