

Solution:

Case 1. The force \vec{F} has so big magnitude that the carts A and B remain at the rest with respect to the cart C , *i.e.* they are moving with the same acceleration as the cart C is. Let \vec{G}_1 , \vec{T}_1 and \vec{T}_2 denote forces acting on particular carts as shown in the Figure 2 and let us write the equations of motion for the carts A and B and also for whole mechanical system. Note that certain internal forces (*viz.* normal reactions) are not shown.

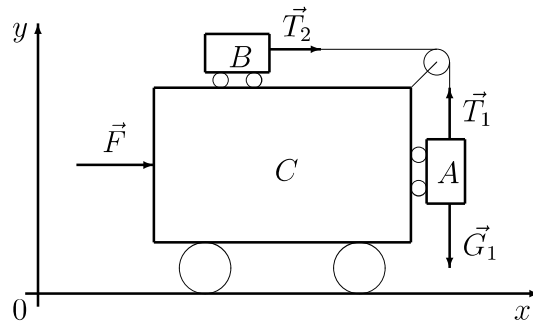


Figure 2:

The cart B is moving in the coordinate system Oxy with an acceleration a_x . The only force acting on the cart B is the force \vec{T}_2 , thus

$$T_2 = m_2 a_x . \quad (1)$$

Since \vec{T}_1 and \vec{T}_2 denote tensions in the same cord, their magnitudes satisfy

$$T_1 = T_2 .$$

The forces \vec{T}_1 and \vec{G}_1 act on the cart A in the direction of the y -axis. Since, according to condition 1, the carts A and B are at rest with respect to the cart C , the acceleration in the direction of the y -axis equals to zero, $a_y = 0$, which yields

$$T_1 - m_1 g = 0 .$$

Consequently

$$T_2 = m_1 g . \quad (2)$$

So the motion of the whole mechanical system is described by the equation

$$F = (m_1 + m_2 + m_3) a_x , \quad (3)$$

because forces between the carts A and C and also between the carts B and C are internal forces with respect to the system of all three bodies. Let us remark here that also the tension \vec{T}_2 is the internal force with respect to the system of all bodies, as can be easily seen from the analysis of forces acting on the pulley. From equations (1) and (2) we obtain

$$a_x = \frac{m_1}{m_2} g.$$

Substituting the last result to (3) we arrive at

$$F = (m_1 + m_2 + m_3) \frac{m_1}{m_2} g.$$

Numerical solution:

$$T_2 = T_1 = 0.3 \cdot 9.81 \text{ N} = 2.94 \text{ N},$$

$$F = 2 \cdot \frac{3}{2} \cdot 9.81 \text{ N} = 29.4 \text{ N}.$$

Case 2. If the cart C is immovable then the cart A moves with an acceleration a_y and the cart B with an acceleration a_x . Since the cord is inextensible (*i.e.* it cannot lengthen), the equality

$$a_x = -a_y = a$$

holds true. Then the equations of motion for the carts A , respectively B , can be written in following form

$$T_1 = G_1 - m_1 a, \tag{4}$$

$$T_2 = m_2 a. \tag{5}$$

The magnitudes of the tensions in the cord again satisfy

$$T_1 = T_2. \tag{6}$$

The equalities (4), (5) and (6) immediately yield

$$(m_1 + m_2) a = m_1 g.$$

Using the last result we can calculate

$$a = a_x = -a_y = \frac{m_1}{m_1 + m_2} g,$$

$$T_2 = T_1 = \frac{m_2 m_1}{m_1 + m_2} g.$$

Numerical results:

$$a = a_x = \frac{3}{5} \cdot 9.81 \text{ m s}^{-2} = 5.89 \text{ m s}^{-2},$$

$$T_1 = T_2 = 1.18 \text{ N}.$$

Problem 2. Water of mass m_2 is contained in a copper calorimeter of mass m_1 . Their common temperature is t_2 . A piece of ice of mass m_3 and temperature $t_3 < 0^\circ\text{C}$ is dropped into the calorimeter.

- Determine the temperature and masses of water and ice in the equilibrium state for general values of m_1 , m_2 , m_3 , t_2 and t_3 . Write equilibrium equations for all possible processes which have to be considered.
- Find the final temperature and final masses of water and ice for $m_1 = 1.00 \text{ kg}$, $m_2 = 1.00 \text{ kg}$, $m_3 = 2.00 \text{ kg}$, $t_2 = 10^\circ\text{C}$, $t_3 = -20^\circ\text{C}$.

Neglect the energy losses, assume the normal barometric pressure. Specific heat of copper is $c_1 = 0.1 \text{ kcal/kg}\cdot^\circ\text{C}$, specific heat of water $c_2 = 1 \text{ kcal/kg}\cdot^\circ\text{C}$, specific heat of ice $c_3 = 0.492 \text{ kcal/kg}\cdot^\circ\text{C}$, latent heat of fusion of ice $l = 78,7 \text{ kcal/kg}$. Take $1 \text{ cal} = 4.2 \text{ J}$.

Solution:

We use the following notation:

t	temperature of the final equilibrium state,
$t_0 = 0^\circ\text{C}$	the melting point of ice under normal pressure conditions,
M_2	final mass of water,
M_3	final mass of ice,
$m'_2 \leq m_2$	mass of water, which freezes to ice,
$m'_3 \leq m_3$	mass of ice, which melts to water.

- Generally, four possible processes and corresponding equilibrium states can occur: