Problem 3. A small charged ball of mass m and charge q is suspended from the highest point of a ring of radius R by means of an insulating cord of negligible mass. The ring is made of a rigid wire of negligible cross section and lies in a vertical plane. On the ring there is uniformly distributed charge Q of the same sign as q. Determine the length l of the cord so as the equilibrium position of the ball lies on the symmetry axis perpendicular to the plane of the ring.

Find first the general solution a then for particular values $Q=q=9.0\cdot 10^{-8}$ C, R=5 cm, m=1.0 g, $\varepsilon_0=8.9\cdot 10^{-12}$ F/m.

Solution:

In equilibrium, the cord is stretched in the direction of resultant force of $\vec{G} = m\vec{g}$ and $\vec{F} = q\vec{E}$, where \vec{E} stands for the electric field strength of the ring on the axis in distance x from the plane of the ring, see Figure 3. Using the triangle similarity, one can write

$$\frac{x}{R} = \frac{Eq}{mg} \,. \tag{11}$$

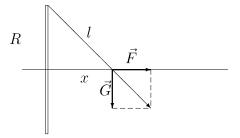


Figure 3:

For the calculation of the electric field strength let us divide the ring to n identical parts, so as every part carries the charge Q/n. The electric field strength magnitude of one part of the ring is given by

$$\Delta E = \frac{Q}{4\pi\varepsilon_0 l^2 n} \,.$$