

## Solution

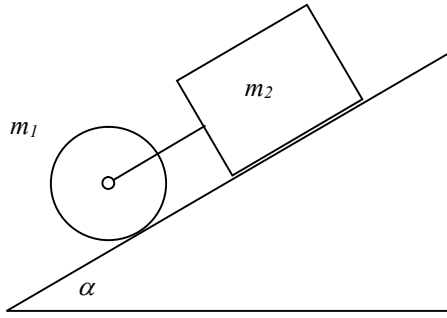


Figure 1

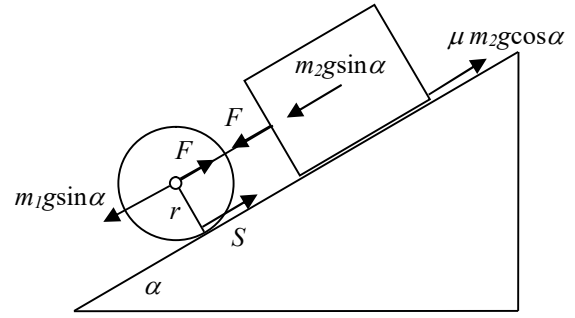


Figure 2

If the cord is stressed the cylinder and the block are moving with the same acceleration  $a$ . Let  $F$  be the tension in the cord,  $S$  the frictional force between the cylinder and the inclined plane (Fig. 2). The angular acceleration of the cylinder is  $a/r$ . The net force causing the acceleration of the block:

$$m_2 a = m_2 g \sin \alpha - \mu m_2 g \cos \alpha + F,$$

and the net force causing the acceleration of the cylinder:

$$m_1 a = m_1 g \sin \alpha - S - F.$$

The equation of motion for the rotation of the cylinder:

$$S r = \frac{a}{r} \cdot I.$$

( $I$  is the moment of inertia of the cylinder,  $S \cdot r$  is the torque of the frictional force.)

Solving the system of equations we get:

$$a = g \cdot \frac{(m_1 + m_2) \sin \alpha - \mu m_2 \cos \alpha}{m_1 + m_2 + \frac{I}{r^2}}, \quad (1)$$

$$S = \frac{I}{r^2} \cdot g \cdot \frac{(m_1 + m_2) \sin \alpha - \mu m_2 \cos \alpha}{m_1 + m_2 + \frac{I}{r^2}}, \quad (2)$$

$$F = m_2 g \cdot \frac{\mu \left( m_1 + \frac{I}{r^2} \right) \cos \alpha - \frac{I \sin \alpha}{r^2}}{m_1 + m_2 + \frac{I}{r^2}}. \quad (3)$$

The moment of inertia of a solid cylinder is  $I = \frac{m_1 r^2}{2}$ . Using the given numerical values:

$$a = g \cdot \frac{(m_1 + m_2) \sin \alpha - \mu m_2 \cos \alpha}{1.5 m_1 + m_2} = 0.3317 g = 3.25 \text{ m/s}^2,$$

$$S = \frac{m_1 g}{2} \cdot \frac{(m_1 + m_2) \sin \alpha - \mu m_2 \cos \alpha}{1.5 m_1 + m_2} = \mathbf{13.01 \text{ N}},$$

$$F = m_2 g \cdot \frac{(1.5 \mu \cos \alpha - 0.5 \sin \alpha) m_1}{1.5 m_1 + m_2} = \mathbf{0.192 \text{ N}}.$$

**Discussion** (See Fig. 3.)

The condition for the system to start moving is  $a > 0$ . Inserting  $a = 0$  into (1) we obtain the limit for angle  $\alpha_1$ :

$$\tan \alpha_1 = \mu \cdot \frac{m_2}{m_1 + m_2} = \frac{\mu}{3} = 0.0667, \quad \alpha_1 = 3.81^\circ.$$

For the cylinder separately  $\alpha_1 = 0$ , and for the block separately  $\alpha_1 = \tan^{-1} \mu = 11.31^\circ$ .

If the cord is not stretched the bodies move separately. We obtain the limit by inserting  $F = 0$  into (3):

$$\tan \alpha_2 = \mu \cdot \left( 1 + \frac{m_1 r^2}{I} \right) = 3\mu = 0.6, \quad \alpha_2 = 30.96^\circ.$$

The condition for the cylinder to slip is that the value of  $S$  (calculated from (2) taking the same coefficient of friction) exceeds the value of  $\mu m_1 g \cos \alpha$ . This gives the same value for  $\alpha_3$  as we had for  $\alpha_2$ . The acceleration of the centers of the cylinder and the block is the same:  $g(\sin \alpha - \mu \cos \alpha)$ , the frictional force at the bottom of the cylinder is  $\mu m_1 g \cos \alpha$ , the peripheral acceleration of the cylinder is  $\mu \cdot \frac{m_1 r^2}{I} \cdot g \cos \alpha$ .

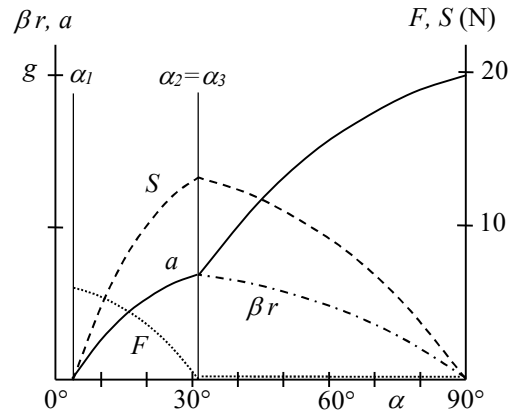


Figure 3