Solution

The devices given to the students allowed using several methods. The students used the following three methods:

1. Comparison of velocity of warming up water and petroleum;
2. Comparison of cooling down water and petroleum;
3. Traditional heat balance.

As no weights were given, the students had to use the sand to find portions of petroleum and water with masses equal to the mass of calorimeter.

First method: comparison of velocity of warming up

If the heater is inside water then both water and calorimeter are warming up. The heat taken by water and calorimeter is:

$$Q_1 = m_w c_w \Delta t_1 + m_c c_c \Delta t_1 ,$$

where: m_w denotes mass of water, m_c - mass of calorimeter, c_w - specific heat of water, c_c - specific heat of calorimeter, Δt_1 - change of temperature of the system water + calorimeter.

On the other hand, the heat provided by the heater is equal:

$$Q_2 = A\frac{U^2}{R}\tau_1 ,$$

where: A – denotes the thermal equivalent of work, U – voltage, R – resistance of the heater, τ_1 – time of work of the heater in the water.

Of course,

$$Q_1 = Q_2 .$$

Thus

$$A\frac{U^2}{R}\tau_1 = m_w c_w \Delta t_1 + m_c c_c \Delta t_1 .$$

For petroleum in the calorimeter we get a similar formula:

$$A\frac{U^2}{R}\tau_2 = m_p c_p \Delta t_2 + m_c c_c \Delta t_2 .$$

where: m_p denotes mass of petroleum, c_p - specific heat of petroleum, Δt_2 - change of temperature of the system water + petroleum, τ_2 – time of work of the heater in the petroleum.

By dividing the last equations we get

$$\frac{\tau_1}{\tau_2}=\frac{m_w c_w \Delta t_1 + m_c c_c \Delta t}{m_p c_p \Delta t_2 + m_c c_c \Delta t_2}.$$

It is convenient to perform the experiment by taking masses of water and petroleum equal to the mass of the calorimeter (for that we use the balance and the sand). For

$$m_w = m_p = m_c$$

the last formula can be written in a very simple form:

$$\frac{\tau_1}{\tau_2}=\frac{c_w \Delta t_1 + c_c \Delta t_1}{c_p \Delta t_2 + c_c \Delta t_2}.$$

Thus

$$c_c = \frac{\Delta t_1}{\tau_1}\frac{\tau_2}{\Delta t_2}c_w - \left(1-\frac{\Delta t_1}{\tau_1}\frac{\tau_2}{\Delta t_2}\right)c_c$$

or

$$c_c = \frac{k_1}{k_2}c_w - \left(1-\frac{k_1}{k_2}\right)c_c,$$

where

$$k_1 = \frac{\Delta t_1}{\tau_1} \quad \text{and} \quad k_2 = \frac{\Delta t_2}{\tau_2}$$

denote "velocities of heating" water and petroleum, respectively. These quantities can be determined experimentally by drawing graphs representing dependence Δt_1 and Δt_2 on time (τ). The experiment shows that these dependences are linear. Thus, it is enough to take slopes of appropriate straight lines. The experimental setup given to the students allowed measurements of the specific heat of petroleum, equal to 0.53 cal/(g°·C), with accuracy about 1%.

Some students used certain mutations of this method by performing measurements at $\Delta t_1 = \Delta t_2$ or at $\tau_1 = \tau_2$. Then, of course, the error of the final result is greater (it is additionally affected by accuracy of establishing the conditions $\Delta t_1 = \Delta t_2$ or at $\tau_1 = \tau_2$).

Second method: comparison of velocity of cooling down

Some students initially heated the liquids in the calorimeter and later observed their cooling down. This method is based on the Newton's law of cooling. It says that the heat Q transferred during cooling in time τ is given by the formula:

$$Q = h(t-\vartheta)s\tau,$$

where: t denotes the temperature of the body, ϑ - the temperature of surrounding, s – area of the body, and h – certain coefficient characterizing properties of the surface. This formula is correct for small differences of temperatures $t-\vartheta$ only (small compared to t and ϑ in the absolute scale).

This method, like the previous one, can be applied in different versions. We will consider only one of them.

Consider the situation when cooling of water and petroleum is observed in the same calorimeter (containing initially water and later petroleum). The heat lost by the system water + calorimeter is

$$\Delta Q_1 = (m_w c_w + m_c c_c)\Delta t ,$$

where Δt denotes a change of the temperature of the system during certain period τ_1. For the system petroleum + calorimeter, under assumption that the change in the temperature Δt is the same, we have

$$\Delta Q_2 = (m_p c_p + m_c c_c)\Delta t .$$

Of course, the time corresponding to Δt in the second case will be different. Let it be τ_2.

From the Newton's law we get

$$\frac{\Delta Q_1}{\Delta Q_2} = \frac{\tau_1}{\tau_2} .$$

Thus

$$\frac{\tau_1}{\tau_2} = \frac{m_w c_w + m_c c_c}{m_p c_p + m_c c_c} .$$

If we conduct the experiment at

$$m_w = m_p = m_c ,$$

then we get

$$c_p = \frac{T_2}{T_1} c_w - \left(1 - \frac{T_2}{T_1}\right) c_c .$$

As cooling is rather a very slow process, this method gives the result with definitely greater error.

Third method: heat balance

This method is rather typical. The students heated the water in the calorimeter to certain temperature t_1 and added the petroleum with the temperature t_2. After reaching the thermal equilibrium the final temperature was t. From the thermal balance (neglecting the heat losses) we have

$$(m_w c_w + m_c c_c)(t_1 - t) = m_p c_p (t - t_2).$$

If, like previously, the experiment is conducted at

$$m_w = m_p = m_c,$$

then

$$c_p = (c_w + c_c)\frac{t_1 - t}{t - t_2}.$$

In this methods the heat losses (when adding the petroleum to the water) always played a substantial role.

The accuracy of the result equal or better than 5% can be reached by using any of the methods described above. However, one should remark that in the first method it was easiest. The most common mistake was neglecting the heat capacity of the calorimeter. This mistake increased the error additionally by about 8%.

Marks

No marking schemes are present in my archive materials. Only the mean scores are available. They are:

Problem # 1	7.6 points
Problem # 2	7.8 points (without the Romanian students)
Problem # 3	5.9 points
Experimental problem	7.7 points

Thanks

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Literature

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[3] **Waldemar Gorzkowski**, *Zadania z fizyki z całego świata (z rozwiązaniami) - 20 lat Międzynarodowych Olimpiad Fizycznych*, WNT, Warszawa 1994 [ISBN 83-204-1698-1]